Pricing Tokens on Industrial Production *

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Abstract

We develop a model for pricing tokens that can be used to get access to industrial production. Our model accounts for the possibility of multiple product lines and the evolution of industrial demand. We apply our model to pricing the ICO of a Swiss startup.

Keywords: ICO, pricing, innovations

1 Introduction

One the key developments in financial technology (FinTech) in the past few years was the rapid growth of new instruments and crowdfunding platforms that facilitate access to funds for small entreprises and, in particular, startups. One such crowdfunding instrument is at initial coin offering (ICO).[1]

In this paper, our goal is to develop a simple model for pricing of a particular type of coins (henceforth, tokens) that give access to future production of goods. Such tokens are special and differ significantly from the more common "security tokens". The key feature that we incorporate into our model is that the nature of services that the users can get access to through their tokens evolves over time due to the process of innovation and economic development. As new product lines emerge over the course of the life of the company, each product line evolves through its own life-cycle, with changing prices and demand. As an example, consider a company producing smartphones. When the first version of the phone (Smartphone 1) appears, it is initially quite expensive; but then, over time, the price starts decreasing, while demand starts

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[1]According to Wikipedia, "In an ICO, a quantity of cryptocurrency is sold in the form of "tokens" ("coins") to speculators or investors, in exchange for legal tender or other cryptocurrencies such as Bitcoin or Ethereum. The tokens sold are promoted as future functional units of currency if or when the ICO's funding goal is met and the project launches." ... Despite their record of failure and the falling prices of cryptocurrencies, a record $7 billion was raised via ICO from January?June 2018

growing. If the company starts producing a new Smartphone 2, this new product line evolves over a similar cycle, with an obvious (negative) impact on prices and quantities sold of Smartphone 1. At the same time, if, instead, the company develops a smart watch, the new product line may have a positive impact on both prices and the demand for smartphones. In the presence of efficiently functioning markets for tokens, the price of a token should incorporate all these economic considerations and reflect both price and demand characteristics for the (multiple) active product lines of the company, as well as expectations about potential future product lines. In this paper, we develop a simple and parsimonious model accounting for all these effects. We illustrate the richness and flexibility of our model by applying it to price a real world ICO of a Swiss startup.

2 Literature Review

While our paper is formally related to pricing crypto assets (see, Nakamoto (2008) and Harvey (2016) for an overview of crypto-finance), there is nothing in our model that is related to blockchain technology. However, effectively, tokens are smart contracts, and blockchain technology can significantly facilitate verification of these contracts; more specifically, for utility tokens, blockchain can be used for verification of issues related to the production activities of the company.[2] See, for example, Yermack (2017), Cao, Cong, and Yang (2018), and Cong and He (2018).

On an organized exchange, the value of the token a be influenced both by the economic fundamentals (which are the topic of this paper) and by the so-called network externalities. That is, the value of the token as a payment devise on an exchange. See, for example, Cong, Li, and Wang (2018), Sockin and Xiong (2018), Li and Mann (2018), Pagnotta and Buraschi (2018). The interaction between fundamental information and transaction value may be non-trivial and is left as a topic for future research. Furthermore, in our model we assume that users have perfect knowledge of all important parameters. Thus, we completely ignore learning effects, which may be significant. See, Athey, Parashkevov, Sarukkai, and Xia (2016).

Catalini and Gans (2018) show how ICOs can be used to fund venture start-up costs. They show that the ICO mechanism allows entrepreneurs to generate buyer competition for the token, which, in turn, reveals consumer value without the entrepreneurs having to know, ex ante, consumer willingness to pay. This is clearly beneficial and opens a road for optimal mechanism and platform design questions. See, for example, Gans and Halaburda (2015), Halaburda and Sarvary (2016), Chiu and Wong (2015), and Chiu and Koeppl (2017). Understanding optimal design of tokens over the life-cycle of a company is another interesting direction for future research.

3 Model

Time is continuous, $t \geq 0$ and uncertainty is modelled by a filtration $\mathcal{F} = (\mathcal{F}_t)_{t \geq 0}$ on the probability space $\mathcal{P}$. At time zero, a company issues tokens (digital coins) that can be used to acquire goods produced by the company in the future. We denote by m_t the supply of tokens at time t. For simplicity, we assume that

[2] Utility tokens in their basic form may or may not be subject to mining, depending on the design of the exchange. In our model, we ignore these considerations. Biais, Bisiere, Bouvard, and Casamatta (2017) and Saleh (2017), Easley, O'Hara, and Basu (2017), Huberman, Leshno, and Moallemi (2017), and Cong, He, and Li (2018) for recent research on this topic.

tokens are only issues at time zero. Tokens used to buy goods are "burned" and disappear after usage. Thus, the supply of tokens, m_t is monotone decreasing over time: $m_{\max} \equiv m_0 \geq m_t$ for all $t \geq 0$.[3]

A token owner may need to use the token for acquiring goods. He derives a private value $u_t,\ t \geq 0$ from it, with u_t being potentially stochastic process adapted to the filtration $\mathcal{F}$. The need for good acquisition arrives at the time τ_u of the first jump of a Poisson process with intensity η_t. We also assume that the exists a token exchange platform on which each token owner can place an ask price, a_t: That is, the price at which he is willing to sell the token. Other market participants who have no (or insufficient amount of) tokens have the possibility to place a bid, b_t, on a platform. Bids are short lived: If a bid is not executed, it disappears.[4] We assume that, upon the arrival of a bid, the ask is executed with a probability given by a function $\mathbf{1}_{a_t \leq b_t}\Pi(a_t, b_t, m_t)$.[5] It is natural to assume that the function Π is monotone decreasing in a_t, and is monotone increasing in b_t. In addition, we assume that Π is monotone decreasing in m_t : The higher the supply of tokens, the higher is the competition among the token-owners for the buyer order flow, and hence the lower is the execution probability for a given ask price a_t.

The key complexity in the problem comes from the possibility of multiple product lines and future product innovations. We denote by $n_t \in \mathbb{Z}_+$ the number of active product lines. For any given product line $\ell = 1, \cdots, n_t$, we assume that buyer order flow, $\mathcal{O}_{\ell,t}$, follows a compound Poisson process with a time-varying intensity $\lambda_{\ell,t} \geq 0$. Upon order arrival, we assume that the bid price, b_t, of the order, as well as the order size, $X_t \in \mathbb{N}$, are drawn from distribution $\xi_\ell(x, b, \theta_{\ell,t})db,\ \ell = 1, \cdots, n_t,\ x \in \mathbb{N}$. Here, $\theta_{\ell,t}$ is a stochastic, time varying parameter on which the distribution of order sizes depends. In addition, existing token-owners burn their tokens according to an exogenously given stochastic process, $\mathcal{Y}_t$, which is a compound Poisson process with intensity γ_t and jump size distribution $\zeta(x),\ x \in \mathbb{N}$. Hence, aggregate token supply dynamics is given by

$$m_t = \max(m_{max} - \mathcal{O}_t - \mathcal{Y}_t,\ 0).$$

Summarizing, the state of the economic system is determined by the state vector,

$$S_t = (u_t, n_t, (\lambda_{1,t}, \theta_{1,t}), \cdots, (\lambda_{n_t,t}, \theta_{n_t,t}),\ \gamma_t),$$

and by the token supply, m_t. We will assume that S_t follows a Markov chain with a finite number of values, $S_1, \cdots, S_\kappa$ and instantaneous transition probabilities $\pi = (\pi_{k,q})_{k,q=1}^{\kappa}$:

$$Prob(S_{t+dt} = S_q | S_t = S_k) = \pi_{k,q}dt\ ,\ k, q = 1, \cdots, \kappa.$$

We assume that the token owner (henceforth, the agent) is risk neutral and discounts future at a rate ρ. The objective of the agent is to select an ask price process, $\mathbf{a} = (a_t,\ t \geq 0)$, that is adapted to $\mathcal{F}_t$ and maximizes the discounted present value of cash flows generated by the token. Thus, the value V_t of a token

[3]Our model can be easily accounted for the case of time-varying supply, whereby the company episodically issues new tokens.

[4]In the application of our model described below, a bid that is not executed is directly matched by the company.

[5]This assumption of a reduced form probability of execution, $P(a_t, b_t, m_t)$ is made for simplicity. It is possible to develop a full blown general equilibrium model that micro-founds this function. Developing such a model would require specifying (heterogeneous) preferences of buyers and sellers and their cross-sectional distribution. We leave this interesting topic for future research.

for the agent is given by

$$V(S_t, m_t) \;=\; \max_{\mathbf{a}} E_t\left[e^{-\rho\tau_u} u_{\tau_u} \mathbf{1}_{\tau_u < \tau_e(\mathbf{a})} \;+\; e^{-\rho\tau_e(\mathbf{a})} b_{\tau_e(\mathbf{a}) \mathbf{1}_{\tau_u > \tau_e(\mathbf{a})}}\right].$$

Here, $\tau_e(\mathbf{a})$ is the *execution time:* The random stopping time at which the ask placed by the agent is executed and the token is burned. Clearly, this stopping time depends on the optimal policy $\mathbf{a}$: the higher are the prices in $\mathbf{a}$, the longer it will take to execute the order.

Conditional on the state being $S_t = S_k$ with $n_k = n(S_t)$ product lines, the expected instantaneous change in the value, $V(S_k, m)$, is given by

$$M(S_k, a, m, V) \;\equiv\; \sum_{\ell=1}^{n_k} \lambda_{\ell,k} \sum_{x\ge 0} \int_{\mathbb{R}} (\Pi(a,b,m)a + (1-\Pi(a,b,m))V(S_k,(m-x)^+) \;-\; V(S_k,m))\xi_\ell(x,b,\theta_{\ell,k})db\,.$$

Indeed, conditional on a order of size x with a bid price b, the supply m jumps to $(m-x)^+$, and the order gets executed with probability $\Pi(a,b,m)$, in which case the user receives a. If the order does not get executed (which happens with probability $1-\Pi(a,b,m)$), then the agent ends up with the value of $V(S_k,(m-x)^+)$.

Standard arguments imply that the value satisfies the Hamilton-Jacobi-Bellman (HJB) equation

$$\begin{aligned}
&-\rho V(S_k,m) \;+\; \eta_k(u_k - V(S_k,m)) \;+\; \sum_q \pi_{k,q}(V(S_q,m) - V(S_k,m)) \\
&+\; \gamma_k \sum_x \zeta(x)(V(S_k,(m-x)^+) \;-\; V(S_k,m)) \;+\; \max_a M(S_k,a,m,V) \;=\; 0\,.
\end{aligned} \tag{1}$$

Thus, at any point in time, the maximization problem of the agent is reduced to finding $\max_a M(S_k,a,m,V)$, which in turn depends on the value V. Denote by

$$\Lambda_k \;\equiv\; \sum_{\ell=1}^{n_k} \lambda_{\ell,k}$$

the total order arrival intensity, aggregated across all product lines. Let also

$$\bar{\pi}_k \;=\; \sum_q \pi_{k,q}$$

is the total instantaneous probability of changing the Markov state. Define the operator $\mathcal{A}: \mathbb{R}^{\kappa\times m_{max}} \to \mathbb{R}^{\kappa\times m_{max}}$ via

$$\begin{aligned}
\mathcal{A}(V)_k \;&=\; (\rho + \eta_k + \bar{\pi}_k + \gamma + \Lambda_k)^{-1}\Bigg(\eta_k u_k \;+\; \sum_q \pi_{k,q} V(S_q,m) \;+\; \gamma_k V(S_k,(m-x)^+) \\
&+\; \max_a M(S_k,a,m,V) + \Lambda_k V(S_k,m)\Bigg)
\end{aligned}$$

Then, standard results imply that this operator is a contraction and hence has a unique fixed point. Hence, the following is true.

Theorem 1 *There exists a unique* $V \in \mathbb{R}^{\kappa \times m_{max}}$ *solving the HJB equation* (1). *The optimal ask price* $a_*(S_k, m)$ *is given by*

$$a_*(S_k, m) \;=\; \arg\max_a M(S_k, a, m, V(S_k, m))\,.$$

4 Application

In this section, we apply the general characterization of Theorem 1 to pricing an ICO of a Swiss startup, LakeDiamond. According to the LakeDiamond white paper,[6] each LakeDiamond token gives access to 1 minute of diamond production. During the ICO, clients (mostly retail investors with no direct interest in using diamonds) had an opportunity to invest in the ICO.

Industrial clients of LakeDiamond willing to produce diamonds need to acquire production time during an over-the-counter (private) negotiation (bargaining) with LakeDiamond. Once the bid price b_t has been negotiated, LakeDiamond submits the buy order to the platform on which ask prices are posted. The buy order is then filled at the posted ask prices, going up from the lowest to the highest ask price, until the required quantity of tokens is acquired. If the number of tokens with ask prices $a_t < b_t$ is smaller than the order size of the industrial client, LakeDiamond immediately fills the un-matched part of the order. Thus, consistent with our assumption above, buy orders are short-lived.

According to the LakeDiamond white paper, they already have one active business line, with the demand coming from mechanical watch producers (line 1). They also anticipate to have three more business lines in the future: Diamond plates for lasers (line 2), plates for transistors (line 3), and plates for biotech (line 4). Importantly, they anticipate that: (i) prices in each business line will be monotonically decreasing over time, while the demand quantities will go up; (ii) at any given moment in time, price per business line is monotone increasing in the business line number. Based on (i) and (ii), we make the following simplifying assumption about the distribution of bid prices:

Assumption 1 *Given the number* n_k *of a active business lines, every state* k *is characterized by a tuple* $((p_1(k), \lambda_1(k)), \cdots (p_{n_k}(k), \lambda_{n_k}(k)), p_{n_k+1}(k))$ *such that:*

(1) $p_1(k) < \cdots < p_{n_k+1}(k)$

(2) prices of orders originating in a business line number ℓ, $\ell = 1, \cdots, n_k$, *are uniformly distributed on the interval* $[p_\ell, p_{\ell+1}]$.

In addition, we will make the following simplifying assumptions:

Assumption 2 *(1) order size always comes a fixed fraction* μ *of* $m_{\max}$ *where* $m_{\max}$ *is proportional to* μ*;*

(2) Execution probability is given by $\Pi(a, b, m) \;=\; \pi_e(m)$ *for some monotone decreasing function* π_e *the taking values in* $[0, 1]$.

[6] See, https://lakediamond.ch/docs/whitepaper.pdf

Under these assumptions, when the state is S_k, a key tradeoff a token-owner is facing is the interval $[p_\ell(k), p_{\ell+1}(k)]$ in which to set the ask price. If the ask a is posted so that $a \in [p_\ell(k), p_{\ell+1}(k)]$, we have

$$M^{(\ell)}(S_k, a, m, V) \equiv \lambda_\ell(k)\Bigg((a - V(S_k, m))\pi^e(m)\frac{p_{\ell+1}(k) - a}{p_{\ell+1}(k) - p_\ell(k)}$$
$$+ \left(1 - \pi^e(m)\frac{p_{\ell+1}(k) - a}{p_{\ell+1}(k) - p_\ell(k)}\right)(V(S_k, m - \mu) - V(S_k, m))\Bigg)$$
$$+ \bar{\lambda}_{\ell+1,n_k}(k)\left(\pi_e(m)(a - V(S_k, m)) + (1 - \pi_e(m))(V(S_k, m - \mu) - V(S_k, m))\right)$$

where we have defined

$$\bar{\lambda}_{\ell+1,n_k}(k) = \lambda_{\ell+1}(k) + \cdots + \lambda_{n_k}(k).$$

The following result follows by direct calculation.

Lemma 2 *We have*

$$\arg\max_{a \in [p_\ell(k), p_{\ell+1}(k)]} M(S_k, a, m, V) = a_*^{(\ell)}(k, V(S_k, m - \mu))$$

where we have defined

$$a_*^{(\ell)}(k, v) = \max\left\{p_\ell(k), \min\left\{\frac{p_{\ell+1}(k) + v}{2} + \frac{(p_{\ell+1}(k) - p_\ell(k))\bar{\lambda}_{\ell+1,n_k}(k)}{2\lambda_\ell(k)}, p_{\ell+1}(k)\right\}\right\}$$

The structure of the optimal ask price is very intuitive. Given his valuation v, token-owner posts a price of $\frac{p_{\ell+1}(k)+v}{2}$ plus a markup proportional to the likelihood of orders coming from product lines with higher numbers. Then, the agent selects the optimal ℓ_* such that

$$\ell_*(V) = \arg\max_\ell M^{(\ell)}(S_k, a_*^{(\ell)}(k, V(S_k, m - \mu)), m, V)$$

and the token value V is determined from the HJB equation (1). Using Theorem 1, we can use standard iteration techniques to solve this equation and then simulate possible sample paths of the token ask prices.

5 Simulation Results

Following the LakeDiamond white paper, we assume that the maximal price level in each of the four possible product lines is give by $p_{\max}(1) = 1$, $p_{\max}(2) = 1.3$, $p_{\max}(3) = 1.8$, $p_{\max}(4) = 2.7$, respectively. Given a state with product line prices $p_1, \cdots, p_{n_k+1}$, we assume that orders from a business line ℓ arrive as a Poisson process with intensity

$$\lambda_\ell(k) = \lambda^* \cdot (10/256) \cdot 1/p_{\ell+1}(k).$$

where in the benchmark parametrization we use $\lambda^* = 1$. For example, when the price level is 1, this corresponds to roughly 10 orders per year in that particular business line, assuming the price level does not change.

We then assume that Markov state transitions happen as follows:

- With a Poisson intensity $\delta = 1/256$ (roughly once a year), each business lines experiences a transition: price falls: $p_{\ell+1} \to p_{\ell+1}\psi$ with $\psi = 0.75$ (that is, once a year price falls by roughly 25%). We assume that there are at most two such transitions possible, so that the lowest $p_{\ell+1}$ possible is $p_{\ell+1} = 0.75^2 p_{\max}(\ell)$.

- If the number of active business lines in $\ell - 1$, then with Poisson intensity of $\nu(\ell)$, the new business line number ℓ arrives. We have chosen $\nu(2) = 0.1/256$, $\nu(3) = 0.003/256$,, and $\nu(4) = 0.001/256$.

- All possible vectors of such prices satisfying the monotonicity constraint $p_1 < \cdots < p_{n_k+1}$ constitute possible Markov states.

- order size is set to $\mu = m_{\max}/100$.

- $\pi_e(m) = \min(m^*/m, 1)$, where in the benchmark parametrization we use $m^* = 30$.

- $\eta = 0.0125/256$ (probability that the customer will use the token to produce his own diamonds is 1.25% per year) and $u = 0.25$ (the private value of diamonds for the customer)

Given this parametrization, we can now study the behaviour of time zero ask prices. Figure 1 shows the dependence of these prices on several key parameters: m^*, capturing the sensitivity of execution probability to the supply of tokens; $\nu(2)$, the instantaneous probability that a second business line opens within the next year; η, the probability that the customer will use the token for producing diamonds; and ρ, the time discount rate. Each of the four sub-plots of Figure 1 shows the behaviour of prices for three values of the order arrival intensity $\lambda^* = 0.1, 0.5$, and 1. Consistent with the intuition, prices increase in λ^*, m^* and $\nu(2)$, while they are decreasing in η and ρ. Indeed, since the value of the token for a customer is assumed to be low, a high η reduces the chances of selling the token to industrial clients at a higher price. Similarly, more impatient token owners (i.e., those with a high ρ) set lower prices to get their trade executed faster. By contrast, an increase in the probability of execution through an increase in λ^* or m^* increases the optimal ask price.

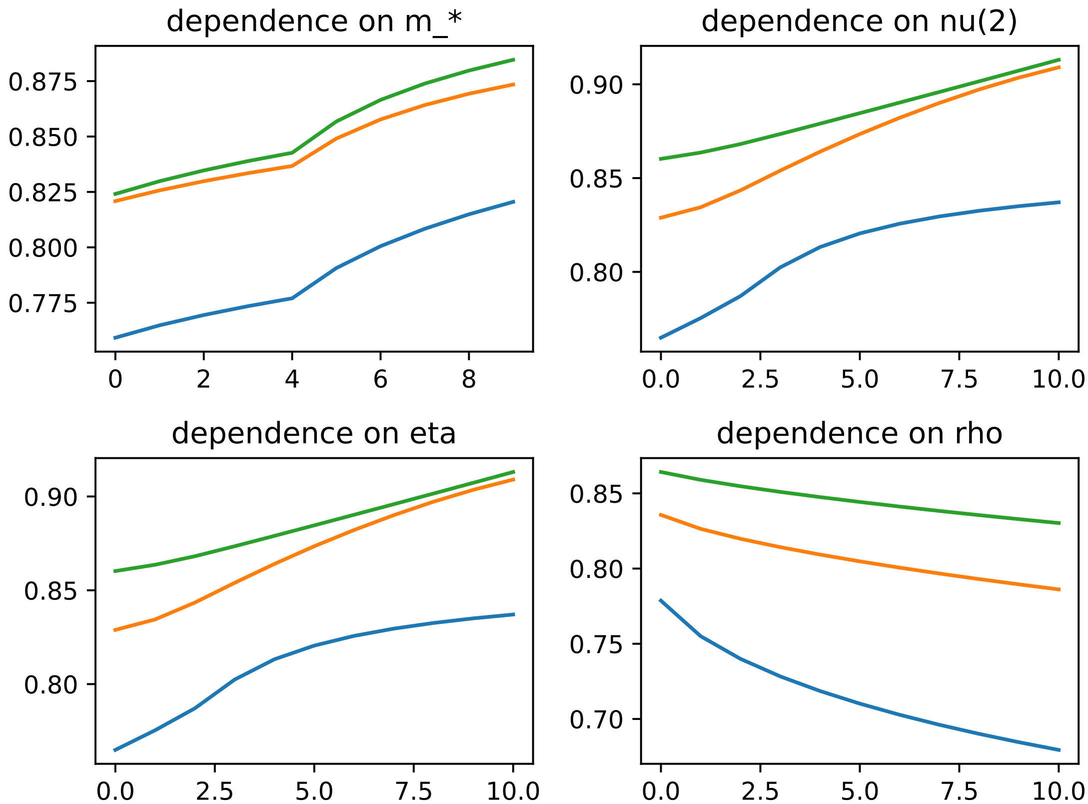

Figure 1: Optimal ask price at time zero. The three lines on each subplot show prices for the three values of $\lambda^* = 0.1, 0.5, 1$. Horizontal axes: 1) dependence on m_*, horizontal axis shows $m_*/3$; 2) dependence on $\nu(2)$, η, ρ: horizontal axis shows the respective variable divided by 0.01. All other variables are fixed at their benchmark levels.

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